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em Economia
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Getulio Vargas

N° 362 ISSN 0104-8910

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Novembro de 1999

URL: http://hdl.handle.net/10438/941

Monteiro de Athayde, Gustavo
Introducing Higher Moments in the CAPM:
Some Basic Ideas/ Gustavo Monteiro de Athayde,
Renato Galvão Flôres Junior - Rio de Janeiro : FGV,EPGE, 2010
(Ensaios Econômicos; 362)

Inclui bibliografia.

CDD-330

INTRODUCING HIGHER MOMENTS IN THE CAPM: SOME BASIC IDEAS*

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Abstract

We show how to include in the CAPM moments of any order, extending the mean-variance or mean-variance-skewness versions available until now. Then, we present a simple way to modify the formulae, in order to avoid the appearance of utility parameters. The results can be easily applied to practical portfolio design, with econometric inference and testing based on generalised method of moments procedures. An empirical application to the Brazilian stock market is discussed.

Keywords: CAPM, GMM, kurtosis, likelihood ratio tests, market portfolio, skewness.

1. Introduction

* We are indebted to the participants in the *Forecasting Financial Markets (London, U.K.)* conferences, since 1997, when these ideas were first discussed. Thanks are also due to Christian Dunis. All mistakes are ours.

Most models in finance are based on mean-variance analysis. The risk premium is therefore derived from the second moment of a random variable. The basic assumption of this kind of modelling is that agents are not concerned about moments higher than the variance. However, it is known that these moments have an influence on investors' decisions, which might explain the bad empirical performance of the CAPM. Indeed, the Lintner-Mossin-Sharpe model[1] is based on the assumption that the investors' goal is to minimise the variance and maximise the expected return of their portfolios.

In general, agents not only care about higher moments, but also their preferences seem to follow some standard behaviour, in which they like odd moments and dislike even ones. Consider, for instance, these two lotteries: the first costs one pound, and there is a chance of $1/10^6$ of winning one million pounds; the second pays one pound upfront, but there exists a probability of $1/10^6$ of having to pay one million pounds. Which one looks more attractive?

What makes this example interesting is that returns from both lotteries have the same mean, variance and even moments. Therefore the difference is on the odd moments. In fact, though the skewnesses of both lotteries have the same absolute value, one is positive and the other negative. The same thing goes with all the higher odd moments.

Most people would prefer the first lottery. Actually it bears the typical profile of existing gambling schemes, like roulettes, horse racing and government lotteries. Individuals are in general willing to trade a highly probable loss of a few cents, for a rather small chance of winning a fortune. Intuition then suggests that agents prefer high positive values for the odd moments.[2]

When the distributions are for instance symmetric, all the odd moments are null. The wider the tails, the higher the even moments will be, and they all basically capture the dispersion of the payoffs. It is also intuitive that agents dislike even moments.

All strictly increasing and concave utility functions have expected utilities that increase with odd moments and decrease with even moments. This kind of behaviour coincides with the aspects

[1] Sharpe (1964) is the classic reference.
[2] One may ask how come that the opposite gambler, who offers these lotteries, is facing negative skewness and still accepting these risks. It should then be reminded that government lotteries, horse racing and casinos always receive some extra premiums, that do not make these bets so fair as in the example above...

just mentioned. Moreover, use of higher moments is the current concern of measures of risk like the VaR (Value at Risk) and the downside risk of portfolios. They all emphasise the worst states of the world, i.e., the left tails of the distributions. The higher the odd moments and the lower the even ones, the lower the risks will be.

One theoretical way to avoid many complications, and get back to mean-variance analysis, is to use the assumption that assets returns are normally distributed. In this case, all the odd moments are null (because the distribution is symmetric) and, since linear combinations of normal distributions are also normal, portfolios made of these assets will exhibit normal returns. As the odd moments are null, optimisation, for the investor, is restricted to minimising the even moments. However, normal distributions give us more; any even moment can be written as:

$$(s^2)^n \prod_{i=1}^{n} (2i-1), \qquad \text{where } 2n \text{ , } n=1,2,\ldots \text{ , is the order of the moment,}$$

so that minimisation of s^2 is sufficient to minimise all the even moments. Thus, the traditional mean-variance problem is justifiable in this context.

Unfortunately, normality of asset returns has been widely rejected in empirical tests. The persistence of skewness has been shown by Singleton and Wingender (1986), and the presence of excess kurtosis in stock returns is widely known. Thus, the previous arguments claim for a more complete model that takes higher moments into account when choosing a portfolio.

Jean (1971, 1973) and Samuelson (1970) approached the optimisation of portfolios taking into account the skewness. Later, Ingersoll (1975) made an attempt at describing what might be a portfolio frontier in three dimensions. Nevertheless, he did not arrive at a closed form for the surfaces, especially because, like in Kraus and Litzenberger (1976), his main concern was the asset pricing relation. The great inconvenience of Ingersoll's and Kraus and Litzenberger's formulae is the presence of preference parameters, as opposed to the original CAPM, that depends solely on observable variables.

In this paper we generalise the CAPM formula to include higher moments, like the kurtosis, and show a way to get rid of preference parameters. The model obtained relies on quantifiable, observable variables. The way to do this is shown in section 2. Section 3 develops empirical tests of the proposal, using data from the Brazilian stock market. A final section concludes.

2. The CAPM with Higher Moments

Ingersoll (1975) and later Kraus and Litzenberger (1976) provided a formula for the CAPM where skewness is taken into account. Both papers assumed that there was an optimal portfolio (which in equilibrium was the market portfolio) and, by total differentiation of the utility function at this point, they arrived at:

$$E(\tilde{r}_i) - r_f = \frac{2U_2 b_{2i} + 3U_3 b_{3i}}{2U_2 + 3U_3} (E(\tilde{r}_m) - r_f)$$

(1)

where $b_{3i} = \frac{s_{im^2}}{s_{m^3}} = \frac{E[(\tilde{r}_i - E(\tilde{r}_i))(\tilde{r}_m - E(\tilde{r}_m))^2]}{E[\tilde{r}_m - E(\tilde{r}_m)]^3}$, $\tilde{r}_i$ is the return of asset i , $\tilde{r}_m$ is the return of the market portfolio, r_f is the riskless rate of return and U_j stands for the marginal utility of the j^{th} moment.

The term s_{im^2} is called the coskewness between asset i and the market portfolio. If it is high and positive, returns $\tilde{r}_i$ tend to go up when the market is turbulent, and to be lower – even below their average - in peaceful, small volatility periods.

In contrast to the CAPM, which does not include utility parameters in its formula, expression (1) requires specification of a utility function in case one wants to estimate its parameters. This might be the reason why, for more than 20 years, formula (1) has not become so popular in financial markets as the CAPM has. Unlike assets returns, preferences are not observable. Thus, we shall be always facing the problem of mispecifying the utility function, when estimating the coefficients.

All estimations of (1) in the literature were made using one of two approaches. The first, Kraus and Litzenberger (1976), Friend and Westerfield (1980), Tan (1991), considered the betas as the independent variables in (1) and used a panel of observations – i.e. return series for different assets. The time dimension was used to estimate, for each asset, the mean return and the corresponding betas; then a linear regression was run in cross-section, taking as observations these estimates.[3] The second approach, Arditti (1971), Francis (1975), starting from the same panel, used a linear regression of each asset's average return on both its variance and skewness; the three moments having been previously obtained from each time series. The intuition behind this regression is to capture how much of the volatility and asymmetry would affect the average return. Needless to say, this last method is totally incoherent with (1) in the sense that we can not check specifically if a given utility function is rejected or not.

Lim (1989) made the first estimation of (1) respecting its original form. He used the generalised method of moments - GMM, and assumed that the marginal rate of substitution between variance and skewness (U_3/U_2) was constant. He then verified that this term was statistically significant. It is immediate to see from (1) that when this term is zero, one is back to the mean-variance CAPM.

A simple generalisation of (1), including higher moments like the kurtosis, can be made (see Athayde and Flôres (1997)) generating:

$$E(\tilde{r}_i) - r_f = \frac{2U_2 b_{2i} + 3U_3 b_{3i} + 4U_4 b_{4i}}{2U_2 + 3U_3 + 4U_4}(E(\tilde{r}_m) - r_f) \quad , \tag{2}$$

where
$$b_{4i} = \frac{s_{im^3}}{s_{m^4}} = \frac{E[(\tilde{r}_i - E(\tilde{r}_i))(\tilde{r}_m - E(\tilde{r}_m))^3]}{E[\tilde{r}_m - E(\tilde{r}_m)]^4} \quad . \tag{3}$$

The term in the numerator of (3) is the cokurtosis and its properties resemble those of the covariance. The only difference is that it captures how asset i responds to the cube of market variations; thus, if positive, it magnifies huge market variations but, unlike the coskewness, it preserves their sign.

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[3] This means that the sample size for the regression was equal to the number of assets.

Equation (2) can be improved in a way to get rid of preference parameters, making the model more easily quantifiable. Consider the three portfolios z_2 , z_3 and z_4 , such that:

$$\begin{array}{lll} s_{mz_2} \neq 0, & s_{m^2z_2} = 0, & s_{m^3z_2} = 0 \\ s_{mz_3} = 0, & s_{m^2z_3} \neq 0, & s_{m^3z_3} = 0 \\ s_{mz_4} = 0, & s_{m^2z_4} = 0, & s_{m^3z_4} \neq 0 \end{array} .$$

The existence of these portfolios is guaranteed if the number of assets is at least equal to the number of moments being considered. The following equations are then immediate and show that if the investor does not care for the j[th] moment (i.e. $U_j=0$), the expected return of portfolio z_j is equal to the riskless asset :

$$E(\tilde{r}_{z_2}) - r_f = \frac{2U_2 b_{z_2}}{2U_2 + 3U_3 + 4U_4}(E(\tilde{r}_m) - r_f) \quad (4)$$

$$E(\tilde{r}_{z_3}) - r_f = \frac{3U_3 b_{z_3}}{2U_2 + 3U_3 + 4U_4}(E(\tilde{r}_m) - r_f) \quad (5)$$

$$E(\tilde{r}_{z_4}) - r_f = \frac{4U_4 b_{z_4}}{2U_2 + 3U_3 + 4U_4}(E(\tilde{r}_m) - r_f) \quad (6)$$

Substituting (4), (5) and (6) in (2), we have:

$$E(\tilde{r}_i) - r_f = b_{2i}\frac{E(\tilde{r}_{z_2}) - r_f}{b_{z_2}} + b_{3i}\frac{E(\tilde{r}_{z_3}) - r_f}{b_{z_3}} + b_{4i}\frac{E(\tilde{r}_{z_4}) - r_f}{b_{z_4}} . \quad (7)$$

This equation says that the risk premium of a portfolio i is explained by its betas. It reminds us of a multi-factor model, like Ross Arbitrage Price Theory – APT (see Ross (1976)). The interesting aspect is that it provides the factors themselves: each of them represents the effect of the respective moment on the asset risk premium. Indeed, the formula offers an orthogonalized decomposition of the risk premium. In case the betas of portfolio i are null, its expected return will equal that of the riskless asset.

As when asset i is the market portfolio itself, all *b*s are unitary, it is also true that:

$$E(\tilde{r}_m) - r_f = \frac{E(\tilde{r}_{z_2}) - r_f}{b_{z_2}} + \frac{E(\tilde{r}_{z_3}) - r_f}{b_{z_3}} + \frac{E(\tilde{r}_{z_4}) - r_f}{b_{z_4}} \quad , \tag{8}$$

which provides a decomposition of the excess return of the market portfolio into the effects of each single moment. Formula (8) thus enables to identify the role of each moment on the market's risk premium. It also follows that if a portfolio i has all its betas equal to 1, it will have the same expected return as the market portfolio.

Combining (7) and (8) to get rid of portfolio z_2 , we finally arrive at:

$$E(\tilde{r}_i) - r_f = b_{2i}(E(\tilde{r}_m) - r_f) + (b_{3i} - b_{2i})\frac{E(\tilde{r}_{z_3}) - r_f}{b_{z_3}} + (b_{4i} - b_{2i})\frac{E(\tilde{r}_{z_4}) - r_f}{b_{z_4}} \quad .(9)$$

An interesting case, which also serves as a cross-check of the above formulae, is when all returns are normally distributed. Recalling Stein's Lemma (see, for instance, Huang and Litzenberger (1988), chapter 4), which says that if x and y are normally distributed, and *f* is a function of class C^1, then:

$$Cov(x, f(y)) = Cov(x, y)E(f'(x)) \qquad ,$$

and normality of returns implies:

$$s_{im^{t-1}} = Cov[\tilde{r}_i, (\tilde{r}_m - E(\tilde{r}_m))^{t-1}] = Cov(\tilde{r}_i, \tilde{r}_m)(t-1)E(\tilde{r}_m - E(\tilde{r}_m))^{t-1}$$

$$b_{it} = \frac{s_{im^{t-1}}}{s_{m^t}} = \frac{Cov(\tilde{r}_i, \tilde{r}_m)(t-1)E(\tilde{r}_m - E(\tilde{r}_m))^{t-1}}{Cov(\tilde{r}_m, \tilde{r}_m)(t-1)E(\tilde{r}_m - E(\tilde{r}_m))^{t-1}} = \frac{s_{im}}{s_{m^2}} = b_{2i} \qquad ,$$

so that (9) becomes:

$$E(\tilde{r}_i) - r_f = \frac{s_{im}}{s_{m^2}}(E(\tilde{r}_m) - r_f) \qquad .$$

(10)

This result shows that, as mentioned in the introduction, even if the investor cares for higher moments, when returns are normally distributed, it is the classical CAPM that applies.

3. An Empirical Test

We have taken series of daily returns for the ten most liquid Brazilian stocks and constructed the z_js portfolios from them. The IBA - *Índice Brasileiro de Ações* played the role of the market portfolio. This index was chosen mainly because of its diversification. The best approximation to the riskless asset return was given by the future contracts of the interbank deposit rates; these rates are exactly those which guarantee the hedge of interest rates.

The tests were performed on a sample running from January 2nd 1996 to October 23rd 1997, giving a total of 450 observations. Estimation and testing were via the GMM, with the long run covariance matrix estimated according to Newey-West (1987), using a truncation lag of 20.

Four models based on formula (9) were tested. The portfolios z_js were chosen to be the minimum variance portfolios, with short sales allowed, subject to the constraints defining them – i.e. that all the cross-moments with the market portfolio were null, except the one of order j.

The most complete model includes skewness and kurtosis, and is described by the following moment conditions. (All returns mentioned from now on are already excess returns.)

The first three conditions define the expected return, variance and skewness of the market portfolio:

$$E[\tilde{r}_m - M_m] = 0$$

$$E\left[(\tilde{r}_m - M_m)^2 - V_m\right] = 0$$

(11)

$$E\left[(\tilde{r}_m - M_m)^3 - S_m\right] = 0 \qquad .$$

The next three conditions define the *b*s for each of the ten assets used, so that they refer to i = 1, 2,..., 10 :

$$E\left[\tilde{r}_i\tilde{r}_m - \tilde{r}_i M_m - b_{2_i} V_m\right] = 0$$

$$E\left[\tilde{r}_i\tilde{r}_m{}^2 - 2\tilde{r}_i\tilde{r}_m M_m + \tilde{r}_i(M_m - V_m) - b_{3_i} S_m\right] = 0$$

(12)

$$E\left[\tilde{r}_i\tilde{r}_m{}^3+3\tilde{r}_i\tilde{r}_m M_m(M_m-\tilde{r}_m)-\tilde{r}_i(M_m{}^3+S_m)-b_{4_i}(\tilde{r}_m-M_m)^4\right]=0 \qquad ,$$

and the following six to the portfolios z_3 and z_4 , in order to guarantee their properties:

$$E\left[\tilde{r}_{z_3}\tilde{r}_m-\tilde{r}_{z_3}M_m\right]=0$$

$$E\left[\tilde{r}_{z_4}\tilde{r}_m-\tilde{r}_{z_4}M_m\right]=0$$

$$E\left[\tilde{r}_{z_3}\tilde{r}_m{}^2-2\tilde{r}_{z_3}\tilde{r}_m M_m+\tilde{r}_{z_3}(M_m-V_m)-b_{z_3}S_m\right]=0$$

$$E\left[\tilde{r}_{z_4}\tilde{r}_m{}^2-2\tilde{r}_{z_4}\tilde{r}_m M_m+\tilde{r}_{z_4}(M_m-V_m)\right]=0$$

(13)

$$E\left[\tilde{r}_{z_3}\tilde{r}_m{}^3+3\tilde{r}_{z_3}\tilde{r}_m M_m(M_m-\tilde{r}_m)-\tilde{r}_{z_3}(M_m{}^3+S_m)\right]=0$$

$$E\left[\tilde{r}_{z_4}\tilde{r}_m{}^3+3\tilde{r}_{z_4}\tilde{r}_m M_m(M_m-\tilde{r}_m)-\tilde{r}_{z_4}(M_m{}^3+S_m)-br_{z_4}(\tilde{r}_m-M_m)^4\right]=0 \qquad .$$

The final three conditions are the ones that will differ between the models. For the first model, which takes skewness and kurtosis into account, they are given by:

$$E\left[\tilde{r}_{z_3}-M_{z_3}\right]=0$$

$$E\left[\tilde{r}_{z_4}-M_{z_4}\right]=0$$

(14)

$$E\left[\tilde{r}_i - b_{2_i}\tilde{r}_m - \frac{(b_{3_i} - b_{2_i})}{b_{z_3}} M_{z_3} - \frac{(b_{4_i} - b_{2_i})}{b_{z_4}} M_{z_4}\right] = 0 \qquad , \qquad i=1, 2,..., 10 \quad .$$

As ten assets were used, the total number of moment conditions is 51 (=3+10x3+6+2+10x1) and that of parameters 37 (=3+10x3+4), resulting in 14 degrees of freedom.

The second model deals only with skewness. When the kurtosis is irrelevant, Mz_4 will be null. The last moment conditions (14) reduce to:

$$E\left[\tilde{r}_{z_3} - M_{z_3}\right] = 0$$

$$E\left[\tilde{r}_{z_4}\right] = 0$$

(15)

$$E\left[\tilde{r}_i - b_{2_i}\tilde{r}_m - \frac{(b_{3_i} - b_{2_i})}{b_{z_3}} M_{z_3}\right] = 0 \qquad , \ i=1,2,...,10 \qquad .$$

In order to verify the gain of adding kurtosis to a model that already contains skewness, a (GMM) Likelihood Ratio Test is used. This test is based on the difference between the two chi-square values associated with the J-test for each model. Hansen's J-test is a portmanteau procedure to assess the validity of the moment conditions; the test statistic is asymptotically a chi-square whose number of degrees of freedom is equal to the number of conditions less the dimension of the vector of parameters.[4] The difference of such chi-squares, in the (GMM) Likelihood Ratio Test, is also (asymptotically) a chi-square and, in the case at stake, with one degree of freedom.

The third model deals only with kurtosis on the asset pricing. This means that Mz_3 will be null and the set of moment conditions (14) are expressed as:

[4] For more details see Davidson and MacKinnon (1993), chapter 17, or Flôres (1997). The basic reference on the J-test is Hansen (1982).

$$E\left[\tilde{r}_{z_3}\right] = 0$$

$$E\left[\tilde{r}_{z_4} - M_{z_4}\right] = 0$$

(16)

$$E\left[\tilde{r}_i - b_{2_i}\tilde{r}_m - \frac{(b_{4_i} - b_{2_i})}{b_{z_4}} M_{z_4}\right] = 0 \qquad , \quad i=1,\ 2,\ldots,\ 10$$

.

The way to verify the gain of adding skewness to a structure that already considers kurtosis is analogous to the one in the previous case; the only difference is that now the chi-square of the first model is subtracted from the one of the third model instead of the second.

The final set of conditions characterises the classical CAPM. Equations (14) must now take into account that both Mz_3 and Mz_4 will be equal to zero, and the model has 16 degrees of freedom. The gain of including skewness (kurtosis) to the classical CAPM is tested by the difference between the J-values of the last and the second (third) model. To assess the gain of adding both higher moments to the CAPM one takes the difference between the chi-squares of the last and first models. This difference should be statistically significant in a chi-square with 2 degrees of freedom.

The main results are shown in Table 1, displaying the statistics related to the J-test and their respective degrees of freedom. These chi-squares check the validity of the moment conditions, and the null that they are "correct" cannot be rejected in any of the four cases in the Table.

To test the significance of including kurtosis to the classical CAPM, we compare the value of 0.12 (13.19-13.07) with tail abscissae from a chi-square with one degree of freedom. The conclusion is that the gain of adding kurtosis is statistically negligible. On the other hand, the gain of adding skewness to the CAPM is statistically significant at 5%: the chi-square value at 95% is 3.84, and the difference between the statistics of the two models is 4.53 (13.19-8.66).

In a similar way, the gain of adding skewness to a model that contains kurtosis is significant at 5% (4.43=13.07-8.64), while that of adding kurtosis to a model that already has skewness is negligible (0.02=8.66-8.64). Finally, in line with the results of the previous paragraph, it is interesting to note that the gain from moving from the CAPM to the full skewness+kurtosis model is not significant at 5% (the corresponding chi-square abscissa is now 5.99 , while 13.19-8.64=4.55).

Table 1: Values of the J-test for the four models

Model	**d.f.**	**Chi-sq.**
CAPM	**16**	**13,19**
KURT	**15**	**13,07**
SKEW	**15**	**8,66**
BOTH	**14**	**8,64**

We may then conclude that, for the Brazilian stock market, there is an unquestionable gain in adding skewness in the design of portfolios, while kurtosis does not seem to play any significant role.

As a second point, one of the most common complaints on the CAPM is that it tends to overestimate the βs. One interesting aspect of our proposal is that the optimal model (the one that deals with skewness) tends to provide lower estimations of the b_2s compared to those of the CAPM. Therefore, inclusion of the third moment seems to have corrected this inconvenience. Figure 1 below shows this property in a dramatic way. Each point corresponds to a stock, its coordinates being the b_2s estimated by the optimal model and the CAPM. The 45° line leaves no doubt about the improvement.

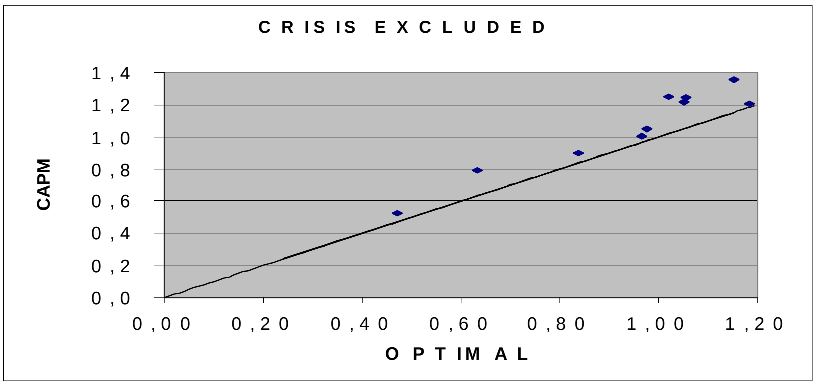

Figure 1: The CAPM and optimal betas for the ten stocks

4. Conclusion

This paper provides a general and applicable way of dealing with the CAPM formula when moments higher than the variance are considered. Moment conditions ensue naturally from the theory and the properties of the instrument portfolios, so that different models can be contrasted through GMM – likelihood ratio tests. The instrument portfolios make it unnecessary to specify a utility function, a stumbling block in the applications of higher moments CAPM-versions until now.

Empirical tests made for the Brazilian market found that skewness played the most important role, while the gain of adding kurtosis was negligible.

Further theoretical developments assessing the validity of additional results of the mean-variance theory are however needed for a complete generalisation of the CAPM.

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